

## Defining Multi-Modal Distributions in Apogee

Apogee is a software tool in the SDI Tools suite that integrates the capabilities of sensitivity analysis, Monte Carlo analysis, allocation, and multi-objective optimization. Apogee works with functions  $Y = f(X)$  that you create freeform in Microsoft Excel™ workbooks, where each X cell is defined as a Parameter that can have statistical variation, and each Y formula cell is defined as a Response. A multi-modal distribution can be assigned to a Parameter in at least five different ways.

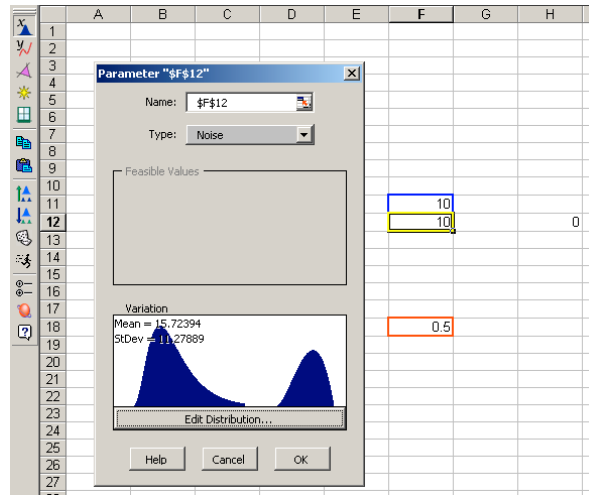


Figure 1 Defining a Parameter with Apogee

**Method 1: Create a composite distribution.** Each parameter in Apogee can be given a *collection* of distribution items. Two examples are shown in Figure 2. On the left is a parameter that has been given three normal distributions with different means, and on the right is a parameter that has been assigned a beta and a truncated lognormal. Both images are snapshots of the Define Distribution window, which is accessed from the Edit Distribution button shown in Figure 1.

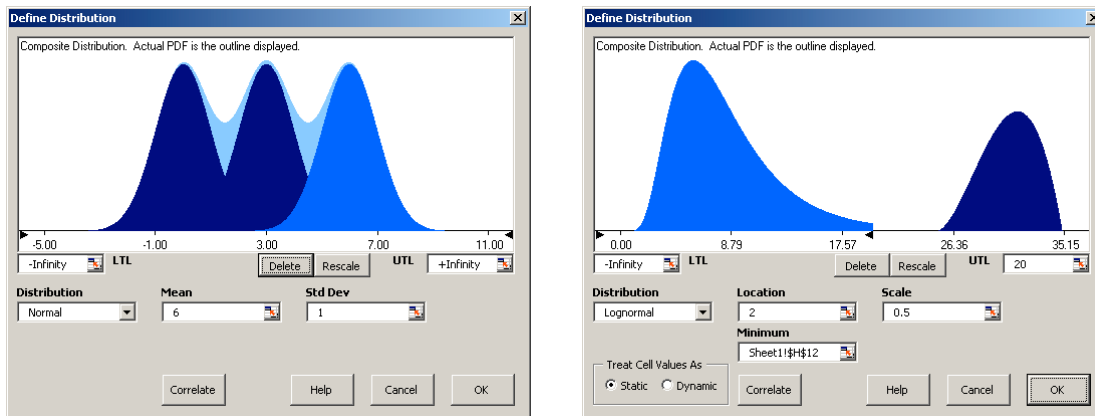


Figure 2 Creating Composite Distributions

Composite distributions are created simply by changing the distribution type or the distribution parameters in the Define Distribution window. Once the changes are made, the “Add” and “Modify” buttons appear below the distribution graph. Clicking the Modify button will modify the active distribution (shown in blue in the graph) to conform to the changed values. Clicking the Add button will add a new distribution with the changed values to the collection. An example is shown in Figure 3, where clicking the Add button will result in the distribution displayed on the left of Figure 2.

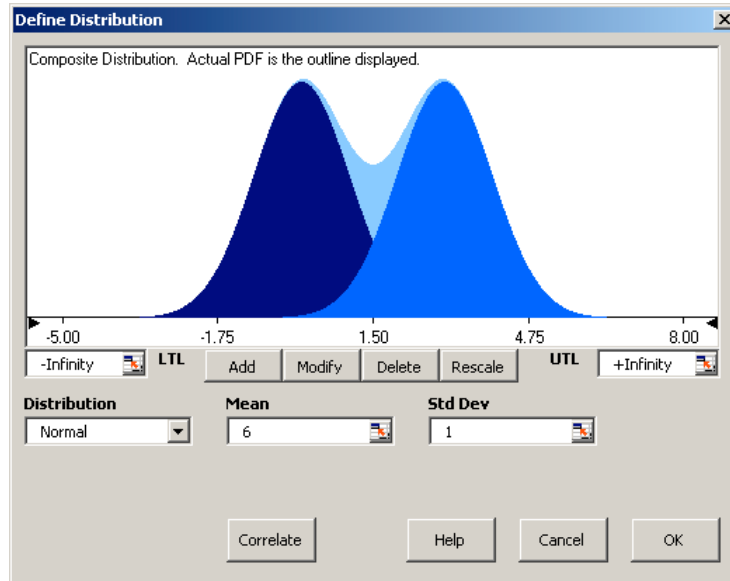


Figure 3 The Add and Modify buttons

**Method 2: Fit a Johnson distribution to data.** The Johnson distribution is actually a flexible *family* of distributions that can be fit to the first four moments (mean, standard deviation, skewness, and kurtosis) of a set of data. If the kurtosis value falls is low enough (typically less than the skewness squared plus 1.8), then a bimodal distribution will result. A bimodal Johnson distribution with a skewness value of zero and a kurtosis value of 1.4 is shown in Figure 4.

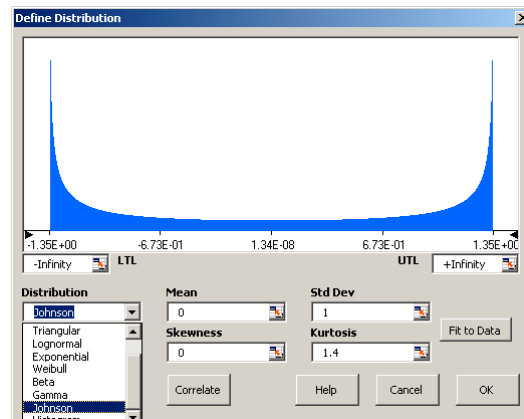


Figure 4 Johnson distribution with a low kurtosis value

**Method 3: Model a set of data as a "spike" histogram.** A parameter’s variation can be specified as a set of real data, defined as two columns – the left column contains the actual values, and the right column contains their frequencies or “counts”. If the data describes multi-modal variation, then the parameter’s

distribution will also be multi-modal. A picture of fitting a “spike” histogram to data and counts is shown in Figure 5.

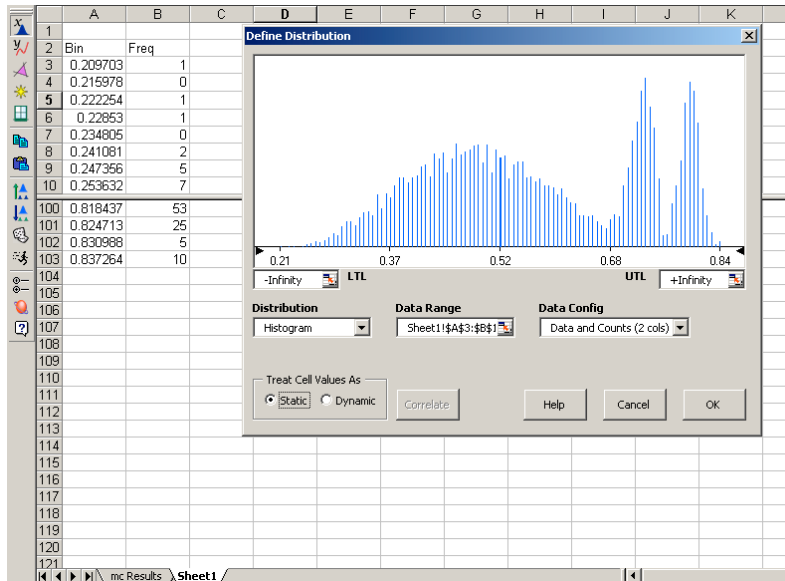


Figure 5 Modeling a set of data as a “spike” histogram

**Method 4: Model a set of data as a "bin" histogram.** A parameter’s variation can be specified as a histogram of real data, defined as three columns – the left column contains the minimum value of each bin, the middle column contains the maximum value of each bin, and the right column contains the bin frequency or “count”. If the data describes multi-modal variation, then the parameter’s distribution will also be multi-modal. A picture of fitting a “bin” histogram to data and counts is shown in Figure 6.

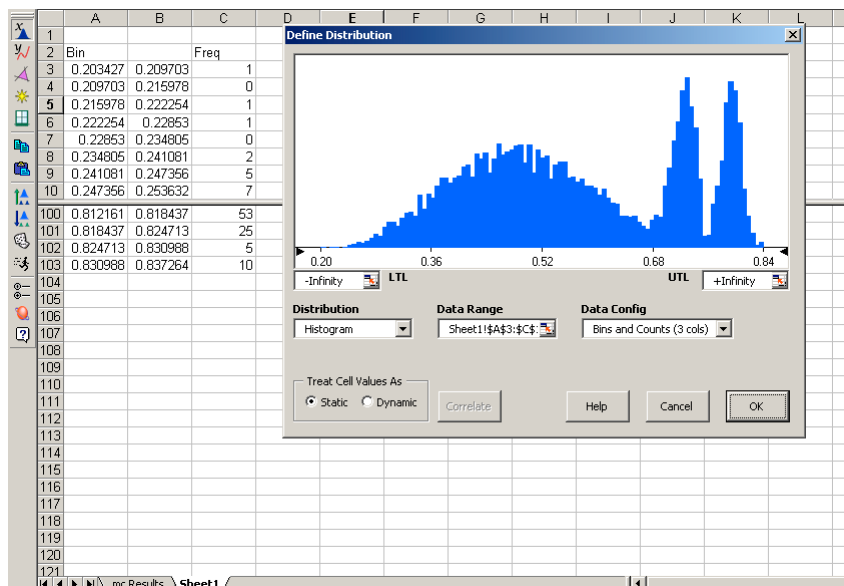
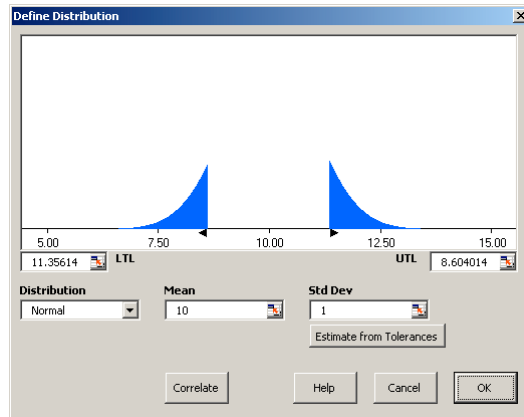


Figure 6 Modeling a set of data as a “bin” histogram

**Method 5: Create a single distribution with “gap” truncation.** One simple way to create a bimodal distribution is to truncate a single distribution such that the lower truncation limit (LTL) is greater than the upper truncation limit (UTL). This will result in the middle of the distribution being truncated, leaving only

the values in the tails of the distribution. Truncation limit values can be changed either by typing values into the LTL and UTL text boxes or by clicking and dragging on the truncation limit sliders ► and ◀ in the distribution graph. An example of a bimodal distribution with this type of truncation is shown in Figure 7.



**Figure 7 A bimodal distribution by using “Gap” truncation**

## Summary

You have seen the ease in which multi-modal distributions can be assigned to parameter variation in Apogee. Once assigned, the effects of this variation on responses can be quickly determined with Sensitivity Analysis and Monte Carlo analysis, and the resulting response variation can be explored and improved with Allocation and Multi-Objective Optimization. To learn more about the statistical capabilities of Apogee, please visit [www.stat-design.com](http://www.stat-design.com).